Lloyd’s mirror effect on signal energy as distance varies

1. Background

This analysis aims to determine the effect of distance between a blast and the hydrophone on the received acoustic energy of the blast signal, purely as a result of the Lloyd’s mirror phenomenon (so excluding the drop in signal energy due to wavefront spreading).

In the classic Lloyd’s mirror in optics, a slit of light placed close to a reflecting surface creates a series of visible fringes in the far field as a result of the interference between the directly propagated light waves arriving at the screen and the waves reflected from the mirror. For approximately monochromatic light this results in a number of well-defined fringes with regular spacing, always starting with a dark fringe at the line of contact between the screen and the mirror surface. Some fringes are also visible for broadband light sources (white light), where the screen shows a dark central fringe.

Figure 1. Lloyd’s mirror fringes for white light (mirror boundary on left). Note dark fringe close to mirror.

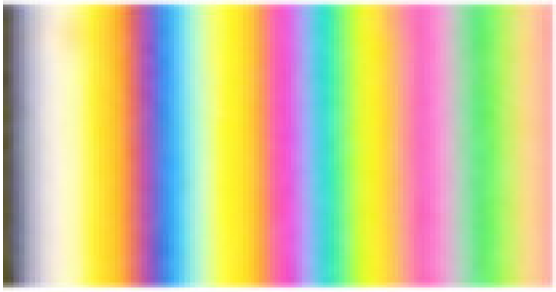

Figure 2: Lloyd’s mirror setup

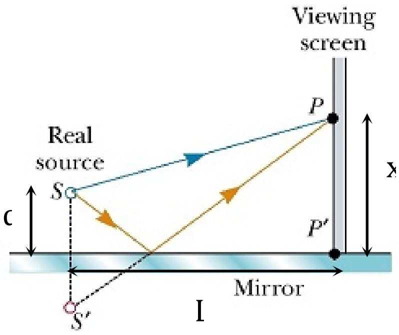

Figure 37.14 Lloyd’s mirror. An interference pattern is produced at point *P* on the screen as a result of the combination of the direct ray (blue) and the reflected ray (red). The reflected ray undergoes a phase change of 180°.

The dark fringe closest to the mirror boundary arises because the light reflected from the mirror undergoes a 180° phase change. At the point P’, the geometric path difference between the source and its reflection is zero, so there is destructive interference at all wavelengths as a result of the 180° phase change of the reflected waves.

Lloyd's mirror theory

The geometrical path difference (GPD) between the point P and the sources S and S' is given by application of Pythagoras' theorem:

Equation 1

In the case where L >> d and x, then the square roots can be approximated using the binomial theorem and keeping only the first term. After simplification we get:

Equation 2

The phase difference, between the interfering waves is given by (where the wavelength of the light is accounting for the phase change on reflection):

Equation 3

The light intensity distribution, I on the screen (the fringes) is defined as follows:

Equation 4

where k is constant linked to the intensity of the light source and geometrical factors. (Note the intensity is at a maximum when the GPD (given by equation 2) is $\lambda/2$, corresponding to constructive interference of the two waves.)

There is a one-to-one correspondence between Lloyd's mirror in optics and the propagation of sound waves from a fish bomb below the water surface. In particular the sound wave reflected from the water surface back into the water column undergoes a similar phase inversion, with the result that the shallow water region at typical detection ranges exhibit suppressed signals due to the mostly destructive interference.

It is useful to give an example that demonstrates this. If the fish bomb explodes at a depth of 5 m (corresponding to the variable d), and the sensor is located at a distance of 5 km (corresponding to the variable L), then the sensor would have to be located at a depth of 125 m for sound waves at a frequency of 3 kHz (a typical modal frequency of a blast wave at distance) to be interfering constructively. Typically, the hydrophones are positioned at a depth of a few metres or so, so they are very much in the region where the sound waves interfere destructively.

With the detector located near the central ‘dark fringe’ zone, then the term in brackets in equation 4 is a small angle and we can use the small angle approximation for the sin function. Thus:

Equation 5

The intensity of the received signal thus varies as the inverse square of the distance, L between the sensor and the bomb (given the bomb depth, d, doesn’t vary, nor the sensor depth, x). This is true for all the acoustic wavelengths of interest in the signal, hence the overall signal energy must be proportional to the inverse square of distance, L.

3. Conclusion

When the sea surface state is relatively flat on the scale of the wavelength of sound waves associated with blasts (centered around 0.5 m) and therefore conducive to their efficient reflection, then the Lloyd’s mirror effect will mean that the energy of a blast signal will vary as the inverse square of the distance to the blast.

It must be emphasized that this inverse square relationship has nothing to do with the natural reduction of signal energy due to spherical or hemispherical spreading. Hemispherical spreading also results in an inverse square relationship of signal energy with detection distance.

Overall, in cases where Lloyd’s mirror applies (calm days with a relatively flat sea surface) , we may expect the energy of a blast signal to vary inversely as the 4th power of the distance to the blast due to hemispherical spreading and Lloyd’s mirror acting in combination.